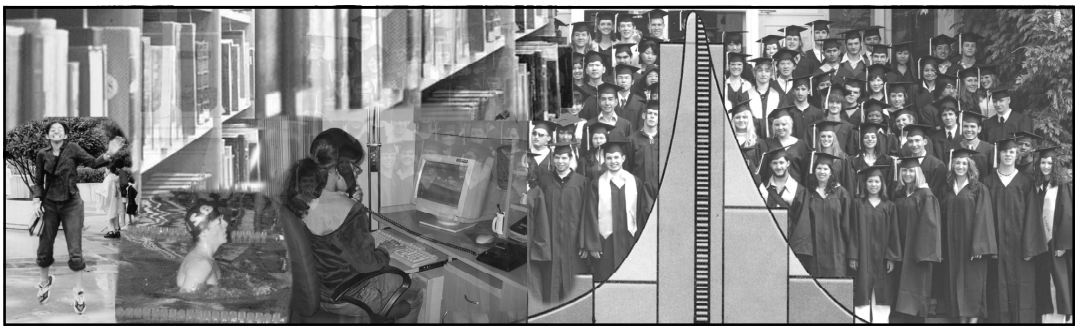
CHAPTER 3

Organisation of Data



Studying this chapter should enable you to:

- *classify the data for further statistical analysis;*
- *distinguish between quantitative and qualitative classification;*
- *prepare a frequency distribution table;*
- *know the technique of forming classes;*
- *be familiar with the method of tally marking;*
- *differentiate between univariate and bivariate frequency distributions.*

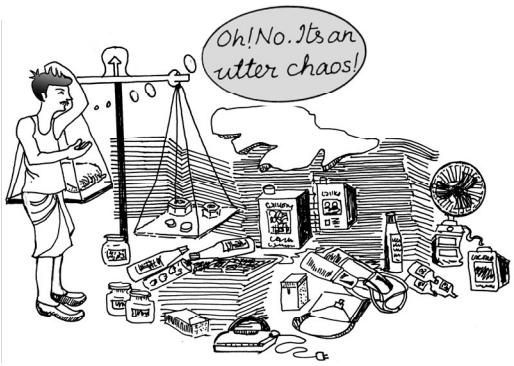
1. INTRODUCTION

In the previous chapter you have learnt about how data is collected. You also came to know the difference between census and sampling. In this chapter, you will know how the data, that you collected, are to be classified. The purpose of classifying raw data is to bring order in them so that they can be subjected to further statistical analysis easily.

Have you ever observed your local junk dealer or *kabadiwallah* to whom you sell old newspapers, broken household items, empty glass bottles, plastics etc? He purchases these things from you and sells them to those who recycle them. But with so much junk in his shop it would be very difficult for him to manage his trade, if he had not organised them properly. To ease his situation he suitably groups or "classifies" various junk. He puts old newspapers together and

ties them with a rope. Then collects all empty glass bottles in a sack. He heaps the articles of metals in one corner of his shop and sorts them into groups like "iron", "copper", "aluminium", "brass" etc., and so on. In this way he groups his junk into different classes — "newspapers, "plastics", "glass", "metals" etc. — and brings order in them. Once his junk is arranged and classified, it becomes easier for him to find a particular item that a buyer may demand.

Likewise when you arrange your schoolbooks in a certain order, it becomes easier for you to handle them. You may classify them



according to subjects where each subject becomes a group or a class. So, when you need a particular book on history, for instance, all you need to do is to search that book in the group "History". Otherwise, you would have to search through your entire collection to find the particular book you are looking for.

While classification of objects or things saves our valuable time and effort, it is not done in an arbitrary manner. The *kabadiwallah* groups his junk according to the markets for reused goods. For example, under the group "Glass" he would put empty bottles, broken mirrors and windowpanes, etc. Similarly when you classify your history books under the group "History" you would not put a book of a different subject in that group. Otherwise the entire purpose of grouping would be lost. *Classification, therefore, is arranging or organising things into groups or classes based on some criteria.*

Activity

- Visit your local post-office to find out how letters are sorted. Do you know what the pin-code in a letter indicates? Ask your postman.

2. RAW DATA

Like the *kabadiwallah's* junk, the unclassified data or *raw data* are highly disorganised. They are often very large and cumbersome to handle. To draw meaningful conclusions from them is a tedious task because they do not yield to statistical methods easily. Therefore proper organisation and presentation of such data is needed before any systematic statistical analysis is undertaken. Hence after collecting data the next step is to organise and present them in a classified form.

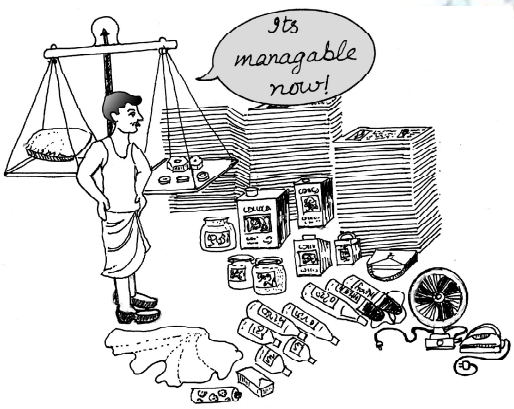
Suppose you want to know the performance of students in mathematics and you have collected data on marks in mathematics of 100

students of your school. If you present them as a table, they may appear something like Table 3.1.

TABLE 3.1
Marks in Mathematics Obtained by 100 Students in an Examination

47	45	10	60	51	56	66	100	49	40
60	59	56	55	62	48	59	55	51	41
42	69	64	66	50	59	57	65	62	50
64	30	37	75	17	56	20	14	55	90
62	51	55	14	25	34	90	49	56	54
70	47	49	82	40	82	60	85	65	66
49	44	64	69	70	48	12	28	55	65
49	40	25	41	71	80	0	56	14	22
66	53	46	70	43	61	59	12	30	35
45	44	57	76	82	39	32	14	90	25

Or you could have collected data on the monthly expenditure on food of 50 households in your neighbourhood to know their average expenditure on food. The data collected, in that case, had you



presented as a table, would have resembled Table 3.2. Both Tables 3.1 and 3.2 are *raw* or *unclassified data*. In both the tables you find that numbers are not arranged in any order. Now if you are asked for the highest marks in mathematics

TABLE 3.2
Monthly Household Expenditure (in Rupees) on Food of 50 Households

1904	1559	3473	1735	2760
2041	1612	1753	1855	4439
5090	1085	1823	2346	1523
1211	1360	1110	2152	1183
1218	1315	1105	2628	2712
4248	1812	1264	1183	1171
1007	1180	1953	1137	2048
2025	1583	1324	2621	3676
1397	1832	1962	2177	2575
1293	1365	1146	3222	1396

from Table 3.1 then you have to first arrange the marks of 100 students either in ascending or in descending order. That is a tedious task. It becomes more tedious, if instead of 100 you have the marks of a 1,000 students to handle. Similarly in Table 3.2, you would note that it is difficult for you to ascertain the average monthly expenditure of 50 households. And this difficulty will go up manifold if the number was larger — say, 5,000 households. Like our *kabadiwallah*, who would be distressed to find a particular item when his junk becomes large and disarranged, you would face a similar situation when you try to get any information from raw data that are large. In one word, therefore, it is a tedious task to pull information from large unclassified data.

The raw data are summarised, and made comprehensible by classification. When facts of similar characteristics are placed in the same class, it enables one to locate them easily, make comparison, and draw inferences without any difficulty. You

have studied in Chapter 2 that the Government of India conducts Census of population every ten years. About 20 crore persons were contacted in Census 2001. The raw data of census are so large and fragmented that it appears an almost impossible task to draw any meaningful conclusion from them. But when the same data is classified according to gender, education, marital status, occupation, etc., the structure and nature of population of India is, then, easily understood.

The raw data consist of observations on variables. The raw data as given in Tables 3.1 and 3.2 consist of observations on a specific or group of variables. Look at Table 3.1 for instance which contains marks in mathematics scored by 100 students. How can we make sense of these marks? The mathematics teacher looking at these marks would be thinking– How have my students done? How many have not passed? How we classify the data depends upon the purpose we have in mind. In this case, the teacher wishes to understand in some depth– how these students have done. She would probably choose to construct the frequency distribution. This is discussed in the next section.

Activity

- Collect data of total weekly expenditure of your family for a year and arrange it in a table. See how many observations you have. Arrange the data monthly and find the number of observations.

3. Classification of Data

The groups or classes of a classification is done in various ways. Instead of classifying your books according to subjects — "History", "Geography", "Mathematics", "Science", etc. — you could have classified them author-wise in an alphabetical order. Or, you could have also classified them according to the year of publication. The way you want to classify them would depend on your requirement.

Likewise the raw data is classified in various ways depending on the purpose. They can be grouped according to time. Such a classification is known as a **Chronological Classification.** In such a classification, data are classified either in ascending or in descending order with reference to time such as years, quarters, months, weeks, etc. The following example shows the population of India classified in terms of years. The variable 'population' is a **Time Series** as it depicts a series of values for different years.

Example 1

Population of India (in crores)

Year	*Population (Crores)*
1951	35.7
1961	43.8
1971	54.6
1981	68.4
1991	81.8
2001	102.7

In **Spatial Classification** the data are classified with reference to geographical locations such as countries, states, cities, districts, etc. *Example 2* shows the yield of wheat in different countries.



Example 2

Yield of Wheat for Different Countries

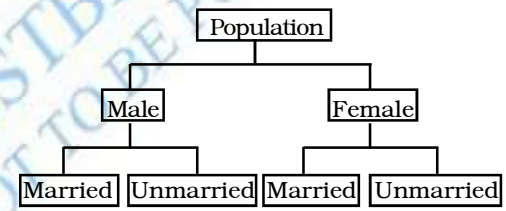
Country	*Yield of wheat (kg/acre)*
America	1925
Brazil	127
China	893
Denmark	225
France	439
India	862

Activities

- In Example 1, find out the years in which India's population was minimum and maximum,
- In Example 2, find the country whose yield of wheat is *slightly more* than that of India's. How much would that be in terms of percentage?
- Arrange the countries of Example 2 in the ascending order of yield. Do the same exercise for the descending order of yield.

Sometimes you come across characteristics that cannot be expressed quantitatively. Such characteristics are called *Qualities or Attributes.* For example, nationality, literacy, religion, gender, marital status, etc. They cannot be measured. Yet these *attributes* can be classified on the basis of either the presence or the absence of a qualitative characteristic. Such a classification of data on attributes is called a *Qualitative Classification.* In the following example, we find population of a country is grouped on the basis of the qualitative variable "gender". An observation could either be a male or a female. These two characteristics could be further classified on the basis of marital status as given below:

Example 3



The classification at the first stage is based on the presence and absence of an attribute i.e. male or not male (female). At the second stage, each class — male and female, is further sub divided on the basis of the presence or absence of another attribute i.e. whether married or unmarried. Characteristics like height, weight, age, income, marks of students, etc. are quantitative in nature. When the collected data of such characteristics are grouped into

Activity

- The objects around can be grouped as either living or non-living. Is it a quantitative classification?

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classes, it becomes a *Quantitative Classification.*

Example 4

Frequency Distribution of Marks in Mathematics of 100 Students

Marks	*Frequency*
0–10	1
10–20	8
20–30	6
30–40	7
40–50	21
50–60	23
60–70	19
70–80	6
80–90	5
90–100	4
Total	*100*

Example 4 shows the quantitative classification of marks in mathematics of 100 students given in Table 3.1.

Activity

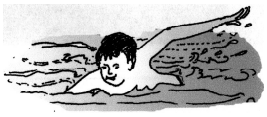
- Express the values of frequency of Example 4 as proportion or percentage of total frequency. Note that frequency expressed in this way is known as *relative frequency*.
- In Example 4, which class has the maximum concentration of data? Express it as percentage of total observations. Which class has the minimum concentration of data?

4. Variables: Continuous and Discrete

A simple definition of variable, which you have read in the last chapter, does not tell you how it varies. Variables differ on the basis of specific criterion. They are broadly classified into *two* types:

(i) *Continuous* and
(ii) *Discrete.*

A *continuous variable* can take *any* numerical value. It may take integral values (1, 2, 3, 4, ...), fractional values (1/2, 2/3, 3/4, ...), and values that are not exact fractions ($\sqrt{2}$ =1.414, $\sqrt{3}$ =1.732, ... , $\sqrt{7}$ =2.645). For example, the height of a student, as he/she grows say from 90 cm to 150 cm, would take all the values in between them. It can take values that are whole numbers like 90cm, 100cm, 108cm, 150cm. It can also take fractional values like 90.85 cm, 102.34 cm, 149.99cm etc. that are not whole numbers. Thus the variable "height" is



capable of manifesting in every conceivable value and its values can also be broken down into infinite gradations. Other examples of a continuous variable are weight, time, distance, etc.

Unlike a continuous variable, a *discrete variable* can take only certain values. Its value changes only by finite "jumps". It "jumps" from one value to another but does not take any intermediate value between them. For example, a variable like the "number of students in a class", for different classes, would assume values that are only whole numbers. It cannot take any

fractional value like 0.5 because "half of a student" is absurd. Therefore it cannot take a value like 25.5 between 25 and 26. Instead its value could have been either 25 or 26. What we observe is that as its value changes from 25 to 26, the values in between them — the fractions are not taken by it. But we should not have the impression that a discrete variable cannot take any fractional value. Suppose X is a variable that takes values like 1/8, 1/16, 1/32, 1/64, ... Is it a discrete variable? Yes, because though X takes fractional values it cannot take any value between two adjacent fractional values. It changes or "jumps" from 1/8 to 1/16 and from 1/16 to 1/32. But it cannot take a value in between 1/8 and 1/16 or between 1/16 and 1/32



Activity

- Distinguish the following variables as continuous and discrete:
  Area, volume, temperature, number appearing on a dice, crop yield, population, rainfall, number of cars on road and age.

Example 4 shows how the marks of 100 students are grouped into classes. You will be wondering as to how we got it from the raw data of Table 3.1. But, before we address this question, you must know what a frequency distribution is.

5. What is a Frequency Distribution?

A frequency distribution is a comprehensive way to classify raw data of a quantitative variable. It shows how different values of a variable (here, the marks in mathematics scored by a student) are distributed in different classes along with their corresponding class frequencies. In this case we have ten classes of marks: 0–10, 10–20, ... , 90–100. The term *Class Frequency* means the number of values in a particular class. For example, in the class 30– 40 we find 7 values of marks from raw data in Table 3.1. They are 30, 37, 34, 30, 35, 39, 32. The frequency of the class: 30–40 is thus 7. But you might be wondering why 40–which is occurring *twice* in the raw data – is not included in the class 30–40. Had it been included the class frequency of 30–40 would have been 9 instead of 7. The puzzle would be clear to you if you are patient enough to read this chapter carefully. So carry on. You will find the answer yourself.

Each class in a frequency distribution table is bounded by *Class Limits.* Class limits are the two ends of a class. The lowest value is called the *Lower Class Limit* and the highest value the *Upper Class Limit.* For example, the class limits for the class: 60–70 are 60 and 70. Its lower class limit is 60 and its upper class limit is 70. *Class Interval or Class Width* is

the difference between the upper class limit and the lower class limit. For the class 60–70, the class interval is 10 (upper class limit *minus* lower class limit).

The *Class Mid-Point* or *Class Mark* is the middle value of a class. It lies halfway between the lower class limit and the upper class limit of a class and can be ascertained in the following manner:

Class Mid-Point or Class Mark = (Upper Class Limit + Lower Class Limit) / 2 .....(1)

The class mark or mid-value of each class is used to represent the class. Once raw data are grouped into classes, individual observations are not used in further calculations. Instead, the class mark is used.

TABLE 3.3
The Lower Class Limits, the Upper Class Limits and the Class Mark

Class	*Frequency*	*Lower Class Limit*	*Upper Class Limit*	*Class Mark*
0–10	1	0	10	5
10–20	8	10	20	15
20–30	6	20	30	25
30–40	7	30	40	35
40–50	21	40	50	45
50–60	23	50	60	55
60–70	19	60	70	65
70–80	6	70	80	75
80–90	5	80	90	85
90–100	4	90	100	95

Frequency Curve is a graphic representation of a frequency distribution. Fig. 3.1 shows the diagrammatic presentation of the frequency distribution of the data in our example above. To obtain the frequency curve we plot the class marks on the X-axis and frequency on the Y-axis.

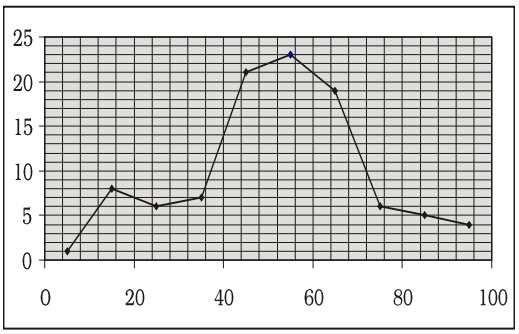


Fig3.1: *Diagrammatic Presentation of Frequency Distribution of Data.*

How to prepare a Frequency Distribution?

While preparing a frequency distribution, the following four questions need to be addressed:

1. How many classes should we have?
2. What should be the size of each class?
3. How should we determine the class limits?
4. How should we get the frequency for each class?

How many classes should we have?

Before we determine the number of classes, we first find out as to what extent the variable in hand changes in value. Such variations in the value of variables are captured by its *range.* The *Range is the difference between*

the largest and the smallest values of variable. A large range indicates that the values of the variable are widely spread. On the other hand, a small range indicates that the values of the variable are spread narrowly. In our example the range of the variable "marks of a student" are 100 because the minimum marks are 0 and the maximum marks 100. It indicates that the variable has a large variation.

After obtaining the value of range, it becomes easier to determine the number of classes once we decide the class interval. Note that *range is the sum of all class intervals.* If the class intervals are equal then range is the product of the number of classes and class interval of a single class.

Range = Number of Classes Class Interval .....(2)

Activities

Find the range of the following:

- population of India in Example 1,
- yield of wheat in Example 2.

Given the value of range, the number of classes would be large if we choose small class intervals. A frequency distribution with too many classes would look too large. Such a distribution is not easy to handle. So we want to have a reasonably compact set of data. On the other hand, given the value of range if we choose a class interval that is too large then the number of classes becomes too small. The data set then may be too compact and we may not like the loss of information about its diversity. For example, suppose the range is 100 and the class interval is 50. Then the number of classes would be just 2 (i.e.100/50 = 2). Though there is no hard-and-fast rule to determine the number of classes, the rule of thumb often used is that the number of classes should be between 5 and 15. In our example we have chosen to have 10 classes. Since the range is 100 and the class interval is 10, the number of classes is 100/10 =10.

What should be the size of each class?

The answer to this question depends on the answer to the previous question. The equation (2) shows that given the range of the variable, we can determine the number of classes once we decide the class interval. Similarly, we can determine the class interval once we decide the number of classes. Thus we find that these two decisions are interlinked with one another. We cannot decide on one without deciding on the other.

In Example 4, we have the number of classes as 10. Given the value of range as 100, the class intervals are automatically 10 by the equality (2). Note that in the present context we have chosen class intervals that are equal in magnitude. However, we could have chosen class intervals that are not of equal magnitude. In that case, the classes would have been of unequal width.

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How should we determine the class limits?

When we classify raw data of a continuous variable as a frequency distribution, we in effect, group the individual observations into classes. *The value of the upper class limit of a class is obtained by adding the class interval with the value of the lower class limit of that class.* For example, the upper class limit of the class 20–30 is 20 + 10 = 30 where 20 is the lower class limit and 10 is the class interval. This method is repeated for other classes as well.

But how do we decide the lower class limit of the first class? That is to say, why 0 is the lower class limit of the first class 0–10? It is because we chose the minimum value of the variable as the lower limit of the first class. In fact, we could have chosen a value less than the minimum value of the variable as the lower limit of the first class. Similarly, for the upper class limit for the last class we could have chosen a value greater than the maximum value of the variable. It is important to note that, when a frequency distribution is being constructed, the class limits should be so chosen that the mid-point or class mark of each class coincide, as far as possible, with any value around which the data tend to be concentrated.

In our example on marks of 100 students, we chose 0 as the lower limit of the first class: 0–10 because the minimum mark was 0. And that is why, we did not choose 1 as the lower class limit of that class. Had we done that we would have excluded the observation 0. The upper class limit of the first class 0–10 is then obtained by adding class interval with lower class limit of the class. Thus the upper class limit of the first class becomes 0 + 10 = 10. And this proce-dure is followed for the other classes as well.

Have you noticed that the upper class limit of the first class is equal to the lower class limit of the second class? And both are equal to 10. This is observed for other classes as well. Why? The reason is that we have used the *Exclusive Method* of classification of raw data. Under the method we form classes in such a way that the lower limit of a class coincides with the upper class limit of the previous class.

The problem, we would face next, is how do we classify an observation that is not only equal to the upper class limit of a particular class but is also equal to the lower class limit of the next class. For example, we find observation 30 to be equal to the upper class limit of the class 20–30 and it is equal to the lower class limit of class 30–40. Then, in which of the two classes: 20–30 or 30–40 should we put the observation 30? We can put it either in class 20–30 or in class 30–40. It is a dilemma that one commonly faces while classifying data in overlapping classes. This problem is solved by the rule of classification in the *Exclusive Method.*

Exclusive Method

The classes, by this method, are formed in such a way that the upper class limit of one class equals the lower class limit of the next class. In this way the continuity of the data is maintained. That is why this method of classification is most suitable in case of data of a continuous variable. *Under the method, the upper class limit is excluded but the lower class limit of a class is included in the interval.* Thus an observation that is exactly equal to the upper class limit, according to the method, would not be included in that class but would be included in the next class. On the other hand, if it were equal to the lower class limit then it would be included in that class. In our example on marks of students, the observation 40, that occurs twice, in the raw data of Table 3.1 is not included in the class 30–40. It is included in the next class 40–50. That is why we find the frequency corresponding to the class 30–40 to be 7 instead of 9.

There is another method of forming classes and it is known as the Inclusive Method of classification.

Inclusive Method

In comparison to the *exclusive method,* the *Inclusive Method* does not exclude the upper class limit in a class interval. It includes the upper class in a class. Thus both class limits are parts of the class interval.

For example, in the frequency distribution of Table 3.4 we include

TABLE 3.4
Frequency Distribution of Incomes of 550 Employees of a Company

Income (Rs)	*Number of Employees*
800–899	50
900–999	100
1000–1099	200
1100–1199	150
1200–1299	40
1300–1399	10
Total	*550*

in the class: 800–899 those employees whose income is either Rs 800, or between Rs 800 and Rs 899, or Rs 899. If the income of an employee is exactly Rs 900 then he is put in the next class: 900–999.

Adjustment in Class Interval

A close observation of the *Inclusive Method* in Table 3.4 would show that though the variable "income" is a continuous variable, no such continuity is maintained when the classes are made. We find *"gap"* or discontinuity between the upper limit of a class and the lower limit of the next class. For example, between the upper limit of the first class: 899 and the lower limit of the second class: 900, we find a "gap" of 1. Then how do we ensure the continuity of the variable while classifying data? This is achieved by making an adjustment in the class interval. The adjustment is done in the following way:

1. Find the difference between the lower limit of the second class and the upper limit of the first class. For example, in Table 3.4 the lower limit of the second class is 900 and

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the upper limit of the first class is 899. The difference between them is 1, i.e. (900 – 899 = 1)

2. Divide the difference obtained in (1) by two i.e. (1/2 = 0.5)
3. Subtract the value obtained in (2) from lower limits of all classes (lower class limit – 0.5)
4. Add the value obtained in (2) to upper limits of all classes (upper class limit + 0.5).

After the adjustment that restores continuity of data in the frequency distribution, the Table 3.4 is modified into Table 3.5

After the adjustments in class limits, the equality (1) that determines the value of class-mark would be modified as the following:

Adjusted Class Mark = (Adjusted Upper Class Limit + Adjusted Lower Class Limit)/2.

TABLE 3.5
Frequency Distribution of Incomes of 550 Employees of a Company

Income (Rs)	*Number of Employees*
799.5–899.5	50
899.5–999.5	100
999.5–1099.5	200
1099.5–1199.5	150
1199.5–1299.5	40
1299.5–1399.5	10
Total	*550*

How should we get the frequency for each class?

In simple terms, *frequency of an observation means how many times that observation occurs in the raw data.* In our Table 3.1, we observe that the value 40 occurs thrice; 0 and 10 occur only once; 49 occurs five times and so on. Thus the frequency of 40 is 3, 0 is 1, 10 is 1, 49 is 5 and so on. But when the data are grouped into

TABLE 3.6
Tally Marking of Marks of 100 Students in Mathematics

Class	*Observations*	*Tally Mark*	*Frequency*	*Class Mark*
0–10	0	/	1	5
10–20	10, 14, 17, 12, 14, 12, 14, 14	𝍸 ///	8	15
20–30	25, 25, 20, 22, 25, 28	𝍸 /	6	25
30–40	30, 37, 34, 39, 32, 30, 35,	𝍸 //	7	35
40–50	47, 42, 49, 49, 45, 45, 47, 44, 40, 44, 49, 46, 41, 40, 43, 48, 48, 49, 49, 40, 41	𝍸 𝍸 𝍸 𝍸 /	21	45
50–60	59, 51, 53, 56, 55, 57, 55, 51, 50, 56, 59, 56, 59, 57, 59, 55, 56, 51, 55, 56, 55, 50, 54	𝍸 𝍸 𝍸 𝍸 ///	23	55
60–70	60, 64, 62, 66, 69, 64, 64, 60, 66, 69, 62, 61, 66, 60, 65, 62, 65, 66, 65	𝍸 𝍸 𝍸 ////	19	65
70–80	70, 75, 70, 76, 70, 71	𝍸 //	6	75
80–90	82, 82, 82, 80, 85	𝍸	5	85
90–100	90, 100, 90, 90	////	4	95
	Total		*100*	

classes as in example 3, the Class Frequency refers to the number of values in a particular class. The counting of class frequency is done by tally marks against the particular class.

Finding class frequency by tally marking

A tally (/) is put against a class for each student whose marks are included in that class. For example, if the marks obtained by a student are 57, we put a tally (/) against class 50 –60. If the marks are 71, a tally is put against the class 70–80. If someone obtains 40 marks, a tally is put against the class 40–50. Table 3.6 shows the tally marking of marks of 100 students in mathematics from Table 3.1.

The counting of tally is made easier when four of them are put as //// and the fifth tally is placed across them as /⁄⁄/. Tallies are then counted as groups of five. So if there are 16 tallies in a class, we put them as /⁄⁄/ /⁄⁄/ /⁄⁄/ / for the sake of convenience. Thus frequency in a class is equal to the number of tallies against that class.

Loss of Information

The classification of data as a frequency distribution has an inherent shortcoming. While it summarises the raw data making it concise and comprehensible, it does not show the details that are found in raw data. There is a loss of information in classifying raw data though much is gained by summarising it as a classified data. Once the data are grouped into classes, an individual observation has no significance in further statistical calculations. In Example 4, the class 20–30 contains 6 observations: 25, 25, 20, 22, 25 and 28. So when these data are grouped as a class 20–30 in the frequency distribution, the latter provides only the number of records in that class (i.e. frequency = 6) but not their actual values. *All values in this class are assumed to be equal to the middle value of the class interval or class mark (i.e. 25). Further statistical calculations are based only on the values of class mark and not on the values of the observations in that class.* This is true for other classes as well. Thus the use of class mark instead of the actual values of the observations in statistical methods involves considerable loss of information. However, being able to make more sense of the raw data as shown more than makes this up.

Frequency distribution with unequal classes

By now you are familiar with frequency distributions of equal class intervals. You know how they are constructed out of raw data. But in some cases frequency distributions with unequal class intervals are more appropriate. If you observe the frequency distribution of Example 4, as in Table 3.6, you will notice that most of the observations are concentrated in classes 40–50, 50–60 and 60–70. Their respective

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frequencies are 21, 23 and 19. It means that out of 100 students, 63 (21+23+19) students are concentrated in these classes. Thus, 63 per cent are in the middle range of 40-70. The remaining 37 per cent of data are in classes 0–10, 10–20, 20–30, 30–40, 70–80, 80–90 and 90–100. These classes are sparsely populated with observations. Further you will also notice that observations in these classes deviate more from their respective class marks than in comparison to those in other classes. But if classes are to be formed in such a way that class marks coincide, as far as possible, to a value around which the observations in a class tend to concentrate, then unequal class interval is more appropriate.

Table 3.7 shows the same frequency distribution of Table 3.6 in terms of unequal classes. Each of the classes 40–50, 50–60 and 60–70 are split into *two* classes. The class 40–50 is divided into 40–45 and 45–50. The class 50–60 is divided into 50– 55 and 55–60. And class 60–70 is divided into 60–65 and 65–70. The new classes 40–45, 45–50, 50–55, 55–60, 60–65 and 65–70 have class interval of 5. The other classes: 0–10, 10–20, 20–30, 30–40, 70–80, 80–90 and 90–100 retain their old class interval of 10. The last column of this table shows the new values of class marks for these classes. Compare them with the old values of class marks in Table 3.6. Notice that the observations in these classes deviated more from their old class mark values than their new class mark values. Thus the new class mark values are more representative of the data in these classes than the old values.

TABLE 3.7
Frequency Distribution of Unequal Classes

Class	*Observations*	*Frequency*	*Class Mark*
0–10	0	1	5
10–20	10, 14, 17, 12, 14, 12, 14, 14	8	15
20–30	25, 25, 20, 22, 25, 28	6	25
30–40	30, 37, 34, 39, 32, 30, 35,	7	35
40–45	42, 44, 40, 44, 41, 40, 43, 40, 41	9	42.5
45–50	47, 49, 49, 45, 45, 47, 49, 46, 48, 48, 49, 49	12	47.5
50–55	51, 53, 51, 50, 51, 50, 54	7	52.5
55–60	59, 56, 55, 57, 55, 56, 59, 56, 59, 57, 59, 55, 56, 55, 56, 55	16	57.5
60–65	60, 64, 62, 64, 64, 60, 62, 61, 60, 62,	10	62.5
65–70	66, 69, 66, 69, 66, 65, 65, 66, 65	9	67.5
70–80	70, 75, 70, 76, 70, 71	6	75
80–90	82, 82, 82, 80, 85	5	85
90–100	90, 100, 90, 90	4	95
	Total	*100*	

Figure 3.2 shows the frequency curve of the distribution in Table 3.7. The class marks of the table are plotted on X-axis and the frequencies are plotted on Y-axis.

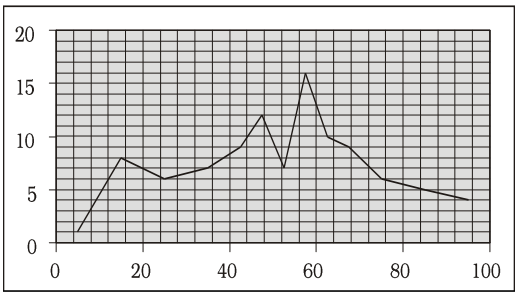

Fig. 3.2: Frequency Curve

Activity

- If you compare Figure 3.2 with Figure 3.1, what do you observe? Do you find any difference between them? Can you explain the difference?

Frequency array

So far we have discussed the classification of data for a continuous variable using the example of percentage marks of 100 students in mathematics. For a discrete variable, the classification of its data is known as a *Frequency Array.* Since a discrete variable takes values and not intermediate fractional values between two integral values, we have frequencies that correspond to each of its integral values.

The example in Table 3.8 illustrates a *Frequency Array.*

TABLE 3.8
Frequency Array of the Size of Households

Size of the Household	*Number of Households*
1	5
2	15
3	25
4	35
5	10
6	5
7	3
8	2
Total	*100*

The variable "size of the household" is a discrete variable that only takes integral values as shown in the table. Since it does not take any fractional value between two adjacent integral values, there are no classes in this frequency array. Since there are no classes in a frequency array there would be no class intervals. As the classes are absent in a discrete frequency distribution, there is no class mark as well.

6. Bivariate Frequency Distribution

The frequency distribution of a single variable is called a Univariate Distribution. The example 3.3 shows the univariate distribution of the single variable "marks of a student". *A Bivariate Frequency Distribution is the frequency distribution of two variables.*

Table 3.9 shows the frequency distribution of two variables, sales and advertisement expenditure (in Rs. lakhs) of 20 companies. The values of sales are classed in different columns

TABLE 3.9
Bivariate Frequency Distribution of Sales (in Lakh Rs) and Advertisement Expenditure (in Thousand Rs) of 20 Firms

	115–125	*125–135*	*135–145*	*145–155*	*155–165*	*165–175*	*Total*
62–64	2	1					3
64–66	1		3				4
66–68	1	1	2	1			5
68–70		2		2			4
70–72		1	1		1	1	4
Total	*4*	*5*	*6*	*3*	*1*	*1*	*20*

and the values of advertisement expenditure are classed in different rows. Each cell shows the frequency of the corresponding row and column values. For example, there are 3 firms whose sales are between Rs 135–145 lakhs and their advertisement expenditures are between Rs 64–66 thousands. The use of a bivariate distribution would be taken up in Chapter 8 on correlation.

7. Conclusion

The data collected from primary and secondary sources are raw or unclassified. Once the data is collected, the next step is to classify them for further statistical analysis. Classification brings order in the data.

The chapter enables you to know how data can be classified through a frequency distribution in a comprehensive manner. Once you know the techniques of classification, it will be easy for you to construct a frequency distribution, both for continuous and discrete variables.

Recap

- Classification brings order to raw data.
- A Frequency Distribution shows how the different values of a variable are distributed in different classes along with their corresponding class frequencies.
- The upper class limit is *excluded* but lower class limit is included in the Exclusive Method.
- Both the upper and the lower class limits are *included* in the Inclusive Method.
- In a Frequency Distribution, further statistical calculations are based only on the class mark values, instead of values of the observations.
- The classes should be formed in such a way that the class mark of each class comes as close as possible, to a value around which the observations in a class tend to concentrate.

EXERCISES

1. Which of the following alternatives is true?

(i) The class midpoint is equal to:
   (a) The average of the upper class limit and the lower class limit.
   (b) The product of upper class limit and the lower class limit.
   (c) The ratio of the upper class limit and the lower class limit.
   (d) None of the above.

(ii) The frequency distribution of two variables is known as
   (a) Univariate Distribution
   (b) Bivariate Distribution
   (c) Multivariate Distribution
   (d) None of the above

(iii) Statistical calculations in classified data are based on
   (a) the actual values of observations
   (b) the upper class limits
   (c) the lower class limits
   (d) the class midpoints

(iv) Under Exclusive method,
   (a) the upper class limit of a class is excluded in the class interval
   (b) the upper class limit of a class is included in the class interval
   (c) the lower class limit of a class is excluded in the class interval
   (d) the lower class limit of a class is included in the class interval

(v) Range is the
   (a) difference between the largest and the smallest observations
   (b) difference between the smallest and the largest observations
   (c) average of the largest and the smallest observations
   (d) ratio of the largest to the smallest observation

2. Can there be any advantage in classifying things? Explain with an example from your daily life.

3. What is a variable? Distinguish between a discrete and a continuous variable.

4. Explain the 'exclusive' and 'inclusive' methods used in classification of data.

5. Use the data in Table 3.2 that relate to monthly household expenditure (in Rs) on food of 50 households and
   (i) Obtain the range of monthly household expenditure on food.
   (ii) Divide the range into appropriate number of class intervals and obtain the frequency distribution of expenditure.
   (iii) Find the number of households whose monthly expenditure on food is
      (a) less than Rs 2000
      (b) more than Rs 3000

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(c) between Rs 1500 and Rs 2500

6. In a city 45 families were surveyed for the number of Cell phones they used. Prepare a frequency array based on their replies as recorded below.

1	3	2	2	2	2	1	2	1	2	2	3	3	3	3
3	3	2	3	2	2	6	1	6	2	1	5	1	5	3
2	4	2	7	4	2	4	3	4	2	0	3	1	4	3

7. What is 'loss of information' in classified data?

8. Do you agree that classified data is better than raw data? Why?

9. Distinguish between univariate and bivariate frequency distribution.

10. Prepare a frequency distribution by inclusive method taking class interval of 7 from the following data.

28	17	15	22	29	21	23	27	18	12	7	2	9	4
1	8	3	10	5	20	16	12	8	4	33	27	21	15
3	36	27	18	9	2	4	6	32	31	29	18	14	13
15	11	9	7	1	5	37	32	28	26	24	20	19	25
19	20	6	9										

Suggested Activity

- From your old mark-sheets find the marks that you obtained in mathematics in the previous class half yearly or annual examinations. Arrange them year-wise. Check whether the marks you have secured in the subject is a variable or not. Also see, if over the years, you have improved in mathematics.